RELIABILITY OF UNDERGROUND PIPELINES
SUBJECT TO CORROSION

By M. Ahammed1 and R. E. Melchers2

(Reviewed by the Pipeline Division)

ABSTRACT: The present paper is concerned with estimating the reliability of underground pipelines subject to externally applied loading and to corrosion, either externally or internally, or both. This affects the strength of the pipeline in resisting the applied loads so that the reliability would be expected to decrease with time. Corrosion is a phenomenon about which there is a great deal of uncertainty. One way of allowing for this is through probabilistic modeling of the material loss as a nonlinear function of time. This approach is explored in the present paper, using a nonlinear function first postulated for atmospheric corrosion. This model is incorporated into expressions for stress resulting from externally applied loading and internal pressure to provide a limit state function expressing the boundary between survival and failure of the system. The probability of system failure is then evaluated using techniques developed for structural reliability analysis.

INTRODUCTION

The design of underground pipelines is governed by the requirement to contain the material being transported, to exclude the materials surrounding the pipe, and to resist externally and internally applied loads. Pressurized pipelines, for example, must resist both internal and external pressures.

For buried pipelines under the action of loads such as traffic loads, it is known that the externally applied stresses acting on the pipe are not uniform on a cross section nor uniform along the length of the pipe. For the present purposes, longitudinal load variation is ignored, that is, support and loading variations along the pipe are assumed to be identical at each cross section. It is assumed also that the pipeline cross section is in a state of plane strain, (i.e., longitudinal movements or deformations are ignored), and that it is at constant and uniform temperature. Again, these assumptions are not unrealistic for buried pipelines.

In addition to the internal and the external pressures, the pipeline is subject, in general, to corrosion, either externally or internally or both. Corrosion affects the pipe-wall thickness. It may be of a uniform nature or localized in extent and severity. It is common practice to coat pipelines externally in an attempt to protect the pipe-wall material from corrosion, but such attempts are not always completely effective, particularly where pipe sections are joined together. Cathodic protection is also used in some cases. Nevertheless, despite these considerable anticorrosion protection efforts, corrosion damage occurs on a large scale and remains a matter of concern due to the costs involved. One of the more difficult aspects of the

1Res. Assoc., Dept. of Civ. Engrg. and Surv., Univ. of Newcastle, NSW 2308, Australia.
2Prof. of Civ. Engrg., Dept. of Civ. Engrg and Surv., Univ. of Newcastle, NSW 2308, Australia.

Note. Discussion open until May 1, 1995. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on April 23, 1993. This paper is part of the Journal of Transportation Engineering, Vol. 120, No. 6, November/December, 1994. ©ASCE, ISSN 0733-947X/94/0006-0989/$2.00 + $.25 per page. Paper No. 5971.
problem is the uncertainty associated with the rate of corrosion and the uncertain location of its occurrence. Because of this, it is appropriate to use a probabilistic approach for the analysis of pipeline reliability. A probabilistic approach provides a quantitative measure of safety and also provides both qualitative and quantitative information about the effects of various uncertain parameters on the safety-measure estimate. This should be of assistance both in reaching proper design decisions and in making decisions about pipeline maintenance and repair strategies. These matters are not considered herein, however.

For pipelines subject to both internal and external loading, an important failure consideration is that of loss of structural strength of the pipeline cross section. Evidently, this is influenced by localized or overall reduction in pipe-wall thickness. Localized loss of wall material arises from pitting and/or crevice corrosion. This is known from experience to be localized, in the sense that only a small part of a pipeline circumference is likely to be affected. Alternatively, such as where protective coating has cracked circumferentially, only a very short local length of pipeline is likely to be affected. In neither of these cases is circumferential pipeline strength likely to be seriously reduced, although perforation of the pipe wall may occur, leading to a serviceability failure (as distinct from the structural failure situation of interest herein).

More important for circumferential strength is loss of wall thickness through general corrosion, which affects much of the circumferential wall thickness uniformly or near so. It is recognized that pipelines may have protective layers—the phenomenon of interest should apply once the protective barrier has lost its effectiveness over a substantial length of pipeline. It should apply also in regions along the pipeline where cathodic protection is less effective than might have been desirable.

WALL STRESSES IN UNDERGROUND PRESSURIZED PIPES

Underground pressured pipelines are subjected to stresses set up by external soil pressure and by internal (fluid) pressure. The effect of these should be given due consideration, although for pressure pipelines the primary stress produced by internal pressure may be of special significance. Internal pressure produces uniform circumferential tension across the wall if the wall thickness is comparatively small and the density of the fluid carried in the pipeline is small relative to the fluid pressure, as is assumed in the present case. The external loads may produce bending stresses both in the longitudinal and circumferential directions. If the pipeline is assumed uniformly loaded and supported along its length, only the circumferential stresses are of interest. The circumferential bending stresses usually are critical either at the top or at the bottom of the pipe. Generally, bending stresses at the two sides of pipe are of a lesser magnitude (Stephenson 1976). The circumferential bending stresses in the pipe wall due to the external loads are assumed to be algebraically additive to the tensile circumferential (i.e., hoop) stress produced by the internal fluid pressure. This is the case provided the pipe wall stresses remain within the elastic range of the material.

The circumferential stress due to internal fluid pressure can be estimated from

\[ s_f = \frac{pr}{t} \]
where it is assumed that \( t << r \) and where \( s_f \) = hoop stress due to internal fluid pressure (MPa); \( p \) = internal fluid pressure (MPa); \( r \) = radius of pipe (mm); and \( t \) = thickness of pipe wall (mm).

The bending stress in the circumferential direction produced in the pipe wall by the external soil loading can be estimated from (Spangler and Handy 1982)

\[
s_s = \frac{6k_mC_d\gamma B_d^2Etr}{Et^3 + 24k_dpr^3}
\]

where \( s_s \) = bending stress due to soil load (MPa); \( C_d \) = calculation coefficient for earth load; \( \gamma \) = unit weight of soil backfill (N/mm\(^3\)); \( B_d \) = width of ditch at the level of top of pipe (m); \( E \) = modulus of elasticity of pipe metal (MPa); \( k_m \) = a bending moment coefficient dependent on the distribution of vertical load and reaction; and \( k_d \) = a deflection coefficient dependent on the distribution of vertical load and reaction.

Further, the circumferential bending stresses produced in the pipe wall due to external traffic loads (such as that resulting from roadway, railway, or airplane traffic) may be estimated from (Spangler and Handy 1982)

\[
s_t = \frac{6k_mI_cC_tFEt r}{A(Et^3 + 24k_dpr^3)}
\]

where \( s_t \) = bending stress due to traffic load (MPa); \( I_c \) = impact factor; \( C_t \) = surface load coefficient; \( F \) = wheel load on surface (N); and \( A \) = effective length of pipe on which load is computed (m).

Provided the pipe wall remains in the elastic range under load, the maximum circumferential tensile stress \( s_m \) is given at the critical sections by

\[
s_m = s_f + s_s + s_t
\]

\[
s_m = \frac{pr}{t} + \frac{6k_mC_d\gamma B_d^2Etr}{Et^3 + 24k_dpr^3} + \frac{6k_mI_cC_tFEt r}{A(Et^3 + 24k_dpr^3)}
\]

CORROSION OF UNDERGROUND PIPELINES

As noted, the loss of pipeline wall thickness due to corrosion may be relatively uniform in extent or it may be localized. However, in neither case does the loss of thickness occur at a constant rate over the design life as is often assumed. The rate of loss of wall thickness is high initially because any corrosion products formed on the surface are porous and have poor protective properties. As is well known, after the initial period, the protective properties of the corrosion products tend to improve. As a result, the corrosion rate usually decreases rather gradually and may even stabilize eventually. It follows that the loss of wall thickness increases with time. Various relationships have been proposed to model corrosion as a function of time. The work in this field is somewhat confused as a result of differences in defining what is meant by “corrosion,” whether it means (1) Weight loss; (2) deepest pit or localized depth; or (3) averaged pit or localized depth. Because the corrosion mainly of interest here is general corrosion, it is assumed that the loss of wall thickness may be modeled empirically by a power law

\[ D = kT^n \]
where \( D \) = loss of wall thickness; \( k \) = a multiplying constant; \( T \) = time of exposure; and \( n \) = an exponential constant. Eq. (5) should be seen as an “engineering” model, rather than one obtained from corrosion “science.” It is based on the model recommended by Kucera and Mattsson (1987) for surface corrosion. By analogy with their work on atmospheric corrosion, and earlier work on corrosion in soil (e.g., Romanov 1957), it follows that for external corrosion, the constants in (5) are very much a function of localized conditions, including the soil type; rate of oxygen depletion and replenishment; nature of soil water or moisture and its movement; and presence and effectiveness of any corrosion protection measures. On the other hand, for internal corrosion, the properties of the fluid being transported are critical, including its interaction with the pipeline material and its potential for physical and/or chemical changes of state as it is being carried along the pipe. However, the available evidence suggests that internally uniform corrosion is less likely to occur under fluid flow conditions. Localized corrosion at the top and/or bottom of the pipeline is more likely.

In many cases it is possible to use past experience to derive estimates for the two constants in (5), but with somewhat more effort than would be necessary to estimate a constant “corrosion rate” as used in conventional corrosion work. Kucera and Mattsson (1987) give some indication of parameters for some typical cases of general corrosion. For corrosion of steel in soil conditions, \( k \) may vary between 0.1 and 0.5 and \( n \) between 0.4 and 1.2.

With the loss of wall thickness given by (5), (1)–(3) become

\[
\text{s}_f = \frac{pr}{t - kT^n}
\]  
\[
\text{s}_g = \frac{6k_mC_d\gamma B_0^2 E r (t - kT^n)}{A (t - kT^n)^3 + 24k_dpr^3}
\]

\[
\text{s}_t = \frac{6k_mC_t F r (t - kT^n)}{A (t - kT^n)^3 + 24k_dpr^3}
\]

RELIABILITY ANALYSIS

It should be evident that in any given situation there is considerable uncertainty associated with the values to be adopted for \( k \) and \( n \). This is particularly the case when no prior, case-specific empirical observations are available. However, even if such information is available, the need to predict future corrosion loss on the basis of past experience leaves some degree of uncertainty about appropriate values to adopt.

The data for many of the other parameters to be inserted into (6)–(8) also is uncertain. Most significant are probably the surface wheel load \( F \) and the impact factor \( I_c \), but the “theoretical” coefficients \( C_t, C_d, k_m \), and \( k_a \) also have uncertainty associated with them, since they need to be selected for a given situation on the basis of limited information. On the other hand, physical quantities such as the modulus of elasticity \( E \), the pipe radius \( r \), and the wall thickness \( t \) generally are subject only to localized variability such as that associated with the pipeline material or with the manufacture of the pipe.

The approach to uncertainty representation to be adopted here is to represent each uncertain variable by a probabilistic description such as a probability density function. In practice, however, the data required to
### TABLE 1. Input for Reliability Calculations of Example Pipeline

<table>
<thead>
<tr>
<th>Symbol (1)</th>
<th>Description</th>
<th>Type (3)</th>
<th>Mean (4)</th>
<th>Coefficient of variation (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_y$</td>
<td>Material yield stress</td>
<td>Normal</td>
<td>450.00 MPa</td>
<td>0.10</td>
</tr>
<tr>
<td>$p$</td>
<td>Internal fluid pressure</td>
<td>Normal</td>
<td>6.205 MPa</td>
<td>0.20</td>
</tr>
<tr>
<td>$r$</td>
<td>Pipe radius</td>
<td>Normal</td>
<td>228.60 mm</td>
<td>0.05</td>
</tr>
<tr>
<td>$t$</td>
<td>Pipe wall thickness</td>
<td>Normal</td>
<td>8.73 mm</td>
<td>0.05</td>
</tr>
<tr>
<td>$k$</td>
<td>Multiplying constant</td>
<td>Normal</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>$n$</td>
<td>Exponential constant</td>
<td>Normal</td>
<td>0.60</td>
<td>0.20</td>
</tr>
<tr>
<td>$k_m$</td>
<td>Bending-moment coefficient</td>
<td>Lognormal</td>
<td>0.235</td>
<td>0.20</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Calculation coefficient</td>
<td>Lognormal</td>
<td>1.32</td>
<td>0.20</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Unit weight of soil</td>
<td>Normal</td>
<td>$18.85 \times 10^{-6}$ N/mm$^3$</td>
<td>0.10</td>
</tr>
<tr>
<td>$B_d$</td>
<td>Width of ditch</td>
<td>Normal</td>
<td>762.00 mm</td>
<td>0.15</td>
</tr>
<tr>
<td>$E$</td>
<td>Modulus of elasticity</td>
<td>Normal</td>
<td>206800.0 MPa</td>
<td>0.05</td>
</tr>
<tr>
<td>$k_d$</td>
<td>Deflection coefficient</td>
<td>Lognormal</td>
<td>0.108</td>
<td>0.20</td>
</tr>
<tr>
<td>$I_c$</td>
<td>Impact factor</td>
<td>Normal</td>
<td>1.5</td>
<td>0.25</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Surface-load coefficient</td>
<td>Lognormal</td>
<td>0.12</td>
<td>0.20</td>
</tr>
<tr>
<td>$F$</td>
<td>Wheel load of traffic</td>
<td>Normal</td>
<td>267000.0 N</td>
<td>0.25</td>
</tr>
<tr>
<td>$A$</td>
<td>Pipe effective length</td>
<td>Normal</td>
<td>914.0 mm</td>
<td>0.20</td>
</tr>
</tbody>
</table>

### TABLE 2. Reliability and Sensitivities as Function of Service Life $T$

<table>
<thead>
<tr>
<th>Symbol (1)</th>
<th>Description</th>
<th>Contribution (%) (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_y$</td>
<td>Material yield stress</td>
<td>$T = 30$ yr (3) $T = 40$ yr (4) $T = 50$ yr (5)</td>
</tr>
<tr>
<td>$p$</td>
<td>Internal fluid pressure</td>
<td>14.97 7.48 5.34</td>
</tr>
<tr>
<td>$r$</td>
<td>Pipe radius</td>
<td>9.94 8.40 7.02</td>
</tr>
<tr>
<td>$t$</td>
<td>Pipe wall thickness</td>
<td>0.21 0.27 0.24</td>
</tr>
<tr>
<td>$k$</td>
<td>Multiplying constant</td>
<td>2.63 2.35 2.03</td>
</tr>
<tr>
<td>$n$</td>
<td>Exponential constant</td>
<td>13.63 18.08 18.55</td>
</tr>
<tr>
<td>$k_m$</td>
<td>Bending-moment coefficient</td>
<td>35.94 56.16 62.29</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Calculation coefficient</td>
<td>4.40 1.39 0.87</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Unit weight of soil</td>
<td>0.15 0.06 0.04</td>
</tr>
<tr>
<td>$B_d$</td>
<td>Width of ditch</td>
<td>0.40 0.01 0.01</td>
</tr>
<tr>
<td>$E$</td>
<td>Modulus of elasticity</td>
<td>0.35 0.13 0.09</td>
</tr>
<tr>
<td>$k_d$</td>
<td>Deflection coefficient</td>
<td>0.22 0.07 0.05</td>
</tr>
<tr>
<td>$I_c$</td>
<td>Impact factor</td>
<td>3.29 1.13 0.72</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Surface-load coefficient</td>
<td>4.09 1.33 0.84</td>
</tr>
<tr>
<td>$F$</td>
<td>Wheel load of traffic</td>
<td>2.91 0.88 0.54</td>
</tr>
<tr>
<td>$A$</td>
<td>Pipe effective length</td>
<td>4.09 1.33 0.84</td>
</tr>
</tbody>
</table>

Note: The reliability indexes for $T = 30$, 40, and 50 yr, respectively, are 1.2716, 0.9671, and 0.7221. Corresponding probabilities of failure are 0.1018, 0.1667, and 0.2351.
define such functions are seldom available. A simpler, and more practical
approach is to use a so-called second-moment description. This defines the
variable mean as the best estimate and its variance or standard deviation
as an estimate of its uncertainty. Simple methods to combine such infor-
mation are given in most elementary books on probability theory. Also, a
more complete second-moment algebra was developed (Ditlevsen 1981),
but this need not be of concern here. Only simple relationships are required.

Methods for estimating structural reliability using probability ideas are
well established (e.g., Melchers 1987; Thoft-Christensen and Baker 1982).
A brief summary relevant to the present problem is given here. In particular,
only the so-called first-order-reliability method is presented.

In the standard problem, all the parameters of the problem that are
uncertain are grouped into the random vector \( \mathbf{X} \). Thus the surface
wheel load \( F \) would be a component \( X_i \) of \( \mathbf{X} \), as would be the corrosion parameters
\( k \) and \( n \). Structural failure occurs when the applied stress exceeds the limiting
or ultimate strength of the pipe-wall material. The applied stress is a function
of some of the \( X_i \), and the strength of the wall a function of some other \( X_i \).
It is then possible to write a function \( Z(X_i) \), the so-called limit state function,
such that positive values of \( Z \) indicate no failure or the safe region, and
negative (or zero) values of \( Z \) indicate failure or the unsafe region. Also,
let \( Z = 0 \) denote the failure “surface.” These are the standard conventions
in structural reliability theory.

Further, let it be assumed, reasonably, that the variables \( X_i \) are mutually
independent and that their mean values \( \bar{X}_i \) and standard deviations \( \sigma(X_i) \)

---

**FIG. 1.** Probability of Failure \( P_f \) versus Service Life \( T \) in Years
are known. The mean value and standard deviation of the limit state function $Z$ can then be approximated by the following expressions:

$$
\bar{Z} \approx Z(\bar{X}_1, \bar{X}_2, \bar{X}_3, \ldots, \bar{X}_n) \tag{9}
$$

$$
\sigma^2(Z) \approx \sum_{i=1}^{N} \left( \frac{\partial Z}{\partial X_i} \sigma(X_i) \right)^2 \tag{10}
$$

when $N$ denotes the total number of design variables. The linearization implicit in (10) is the first-order part of the conventional structural reliability approach—the approximation involved is discussed in the literature [e.g., Melchers (1987)] and is of significance only if the “hypersurface” $Z = 0$ is highly nonlinear in the region of the problem contributing most to the failure probability. It is not a significant matter for the present problem. Also, the derivatives $\frac{\partial Z}{\partial X_i}$ may be evaluated at the point $X = \bar{X}$ using standard techniques described in the literature.

Using conventional rules for manipulating random variables described by means and variances, and by using approximate transformations, the mean $\bar{Z}$ and variance $\sigma^2(Z)$ can be obtained. This means that $Z$ could be considered to be described by a normal distribution (which is sufficiently characterized by a mean and variance). From standard structural reliability theory, the so-called reliability index is then given by

$$
\beta = \frac{\bar{Z}}{\sigma(Z)} \tag{11}
$$
Also, it is a simple matter to evaluate the relative contribution of each $X_i$ to the variance (i.e., uncertainty) of the failure function $Z(X)$. This is expressed as ($\alpha_i^2$), given by

$$\alpha_i^2 = \frac{\left(\frac{\partial Z}{\partial X_i} \sigma(X_i)\right)^2}{\sigma^2(Z)}$$

(12)

The $\alpha_i^2$ term is known as a sensitivity coefficient and, evidently, provides a measure of the importance of $X_i$ to the reliability index $\beta$. It should be noted that the probability of failure $P_f$ is obtained from

$$P_f = P[Z \leq 0] = \Phi \left[ \frac{Z - \bar{Z}}{\sigma(Z)} \right]_{Z=0}$$

(13a)

or

$$P_f = \Phi(-\beta)$$

(13b)

where $\Phi$ = the cumulative distribution function for the standard normal distribution [e.g., Melchers (1987)]; and $\beta$ = the reliability index given by (11). Eq. (13) implies that $\beta$ increases as the probability magnitude decreases.

To apply this approach to underground pipelines, it is necessary to define
the relationship that defines when the pipeline fails, that is, the limit state function $Z(X)$. If the yield strength $s_y$ of the pipe material is taken to be the maximum allowable stress, that is, the stress at which failure occurs, then the limit state function is

$$Z = s_y - \frac{pr}{t - kT^n} - \frac{6k_mC_d\gamma B_dE_r(t - kT^n)}{E(t - kT^n)^3 + 24k_dpr^3} - \frac{6k_mI_cC_iF E_r(t - kT^n)/A}{E(t - kT^n)^3 + 24k_dpr^3}$$

with $Z \leq 0$ indicating failure. Since (14) is nonlinear, an iterative solution technique is required for the calculation of $\alpha_i$ and $\beta$ (Melchers 1987; Thoft-Christensen and Baker 1982).

**EXAMPLE APPLICATION**

Consider the determination of the time-dependent reliability of an underground steel pipe under a roadway. The main purpose of this example is to illustrate the method: the numerical values of variables used are not unrealistic but are nevertheless arbitrary (see Table 1). Some variables, such as the pipe radius and pipe wall thickness have little inherent uncertainty. What there is, is the result of geometrical imperfections such as those which may develop during the process of manufacture. In contrast, some parameters show a high degree of uncertainty. These include the corrosion parameters ($k$ and $n$), which are typically determined from regression analysis.
FIG. 5. Probability of Failure $P_f$ versus $n$ for Various Values of Service Life $T$ [(1) $T = 20$ yr; (2) $T = 25$ yr; (3) $T = 30$ yr]

on observed and experimental data obtained for some specific soil and other conditions. When there is little or no information on which to base a choice for $n$ and/or $k$, known values from other situations might be used, with judicious selection of means and variances to reflect inherent uncertainties. A subjective probability approach may be necessary in some circumstances.

The bending and deflection coefficients $k_m$ and $k_d$ are dependent on the distribution of the load applied across the top of the pipe and on the distribution of the reaction generated across the bottom of the pipe. In practice, it is not usual for the bed to be shaped to fit the contour of the pipe. The width of bottom contact is dependent on the amount by which the pipe pushes into the soil bedding. This is dependent mainly on the vertical load that is transmitted by the pipe to the soil bed and on the stiffness of the soil bed. Both are highly uncertain. A conservative estimate is that the applied load is distributed across the entire width of the pipe top and that the reaction is distributed over a width defined by an about 30° spread across the pipe bottom (Spangler and Handy 1982). Clearly there is a degree of uncertainty associated with these assumptions—this is reflected here in the values selected for the variances for $k_m$ and $k_d$.

Table 1 summarizes the values of the means and coefficients of variation and the probability distribution for each of the variables. Note that the coefficient of variation (COV) is defined as the ratio of the standard deviation to the mean.

The choice of distribution shown in Table 1 is based on the reasoning that where no information is available about an appropriate distribution,
FIG. 6. Probability of Failure $P_f$ versus $k_m$ (Bending-Moment Coefficient) for Various Values of Service Life $T$ [(1) $T = 0$ yr; (2) $T = 10$ yr; (3) $T = 20$ yr; (4) $T = 30$ yr; (5) $T = 40$ yr; (6) $T = 50$ yr]

the normal distribution was selected. It is likely that in a practical application, such knowledge may be available—it can be incorporated readily in available reliability analysis programs (Melchers 1987). Lognormal distributions were chosen for those parameters for which both negative values are not admissible physically, and such values would have been likely if the normal distribution had been applied. As in conventional structural reliability theory, the central-limit theorem and the linearization of (9) and (10) approximates $Z$ as normal.

Some results of the reliability calculation as a function of service life are given in Table 2. The reliability indices and the associated probabilities of failure are shown at the foot of the table for different service lives. The body of the table shows the sensitivities $\alpha^2$, which represent the relative contributions of the variables to $\beta$ or $P_f$. It can be observed that the relative contribution of some of the variables is very low, and that they remain low for all values of service life, indicating that their uncertainty characteristics have little influence on the overall reliability estimate. Hence, it is possible to treat these variables as deterministic quantities in any future analysis without introducing significant error in the estimated probability of failure (or the reliability index $\beta$). Significantly, the relative contribution of the corrosion parameters $k$ and $n$ increases appreciably with increased service life. This confirms the expectation that the corrosion parameters are very important for the design of underground pipelines with long design lives $T$. 

999
Also, the results show that the probability of failure roughly doubles in increasing the service life from 30 years to 50 years.

Fig. 1 shows the manner in which the failure probability changes with increased service life $T$ for the particular case of all random variables taking the values given in Table 1. It is evident that there is a steady increase in probability of failure in the earlier years, but that this increases markedly after about 25 years. The reason for this becomes evident in the following.

Fig. 2 shows the relative contribution of the corrosion parameter $k$ [see (5)] on the variance of the failure function, again with all other variables (Table 1) kept constant. The contribution of $k$ to the probability of failure $P_f$ increases with time and the relationship is nonlinear. For low values of service life (say, less than 20 years), the contribution of $k$ is not very significant, but for high values of service life (say, more than 35 years), the contribution of $k$ is quite significant. This is as expected, since for high values of service life there is a high degree of corrosion penetration.

The relative contribution of the corrosion parameter $n$ on the variance of the failure function was investigated in a similar manner (see Fig. 3). Evidently increasing the service life increases the contribution of $n$, as expected.

Figs. 2 and 3 indicate the important roles of $k$ and $n$ in the reliability assessment, so that an understanding of their effect on the probability of failure might be useful. This is demonstrated in Figs. 4 and 5 as a function of service life. In Fig. 4, except for $k$, all parameters were kept at the values
given in Table 1. The results show a general tendency for probability of failure to increase with increases in the value of $k$. This is as expected, since as $k$ increases wall thickness decreases, and hence the magnitude of circumferential stress increases so that the probability of failure increases. A generally similar pattern is seen when $n$ is varied (see Fig. 5).

The other parameters about which there is significant uncertainty are $k_m$ and $k_d$. Fig. 6 shows the relationship between probability of failure and bending moment coefficient $k_m$ for several values of service life. Within the range of reasonable $k_m$ values [see, e.g., Stephenson (1976)], the probability of failure increases with increasing $k_m$ for all values of service life. Of course, this is to be expected because higher values of $k_m$ give higher values of bending moment and stress, and hence a higher failure probability. In Fig. 7, $k_d$ is varied from 0.08 to 0.14, which is more or less the range reported in the literature [e.g., Stephenson (1976)]. The probability of failure decreases with increases in the value of $k_d$.

PRACTICAL IMPLICATIONS

The results show that for reasonably typical values of the mean and variance of the parameters entering the problem as defined, the corrosion parameters $n$ and $k$ [see (5)] are by far the most critical in estimating the probability of failure of the pipeline under traffic loading (see Table 2). The individual effect of $k$ is shown in Fig. 4. Similarly the individual effect of $n$ is shown in Fig. 5.

Typical values of $(k, n)$ for unprotected steel pipes might be $(0.3, 0.6)$. For a service life of 20 years it is seen from Fig. 4 that $P_f = 0.07$. If $k$ were to change to 0.35, $P_f$ becomes 0.085, a 21% increase. Similarly, changing $n$ from 0.6 to 0.65 changes $P_f$ from 0.07 to 0.085, also a 21% increase. The significance of these observations is that in practical terms, the probability of failure is reasonably sensitive to both $k$ and $n$ selected at reasonably practical values. Hence, considerable effort should be made in attempts to obtain good estimates of $k$ and $n$ in (5).

Finally, if the pipeline is coated and there is sufficient care taken with the coating, it might be expected that no corrosion will occur prior to the deterioration of the coating. If the coating integrity has a life of, say, $C$ years, the service life curve of Fig. 1 simply shifts $C$ years to the right. More generally, however, the coating also will have a probability of failure that is likely to be nonzero. The probabilistic characteristics and hence the probability of coating breakdown could be determined in a manner directly analogous to the theory presented herein.

CONCLUSIONS

In the present paper, a method was presented for the estimation of the structural reliability of underground pipes under the action of both external and internal loading, and incorporating the effect of corrosion. Uncertainties involved in material and soil properties, internal and external loads, and corrosion parameters are considered through the use of probability theory.

From a numerical investigation of an example pipeline, it was found that there is a very significant long-term contribution of the corrosion parameters to structural reliability deterioration, and that this is directly related to the corrosion parameters $(k$ and $n$). The results also show that at low service-life values the reliability index estimate is very sensitive to the changes of the moment and deflection coefficients $(k_m$ and $k_d)$. 

1001
APPENDIX I. REFERENCES


APPENDIX II. NOTATION

The following symbols are used in this paper:

\[ A = \text{pipe effective length}; \]
\[ B_d = \text{width of ditch}; \]
\[ C_d = \text{calculation coefficient for earth load}; \]
\[ C_t = \text{surface-load coefficient}; \]
\[ D = \text{wall-thickness loss}; \]
\[ E = \text{modulus of elasticity of pipe metal}; \]
\[ F = \text{surface wheel load}; \]
\[ I_c = \text{impact factor}; \]
\[ k = \text{multiplying constant}; \]
\[ k_d = \text{deflection coefficient}; \]
\[ k_m = \text{bending-moment coefficient}; \]
\[ N = \text{number of design variables}; \]
\[ n = \text{exponential constant}; \]
\[ P_f = \text{probability of failure}; \]
\[ p = \text{internal fluid pressure}; \]
\[ r = \text{pipe radius}; \]
\[ s_f = \text{hoop stress due to internal fluid pressure}; \]
\[ s_m = \text{maximum circumferential tensile stress}; \]
\[ s_s = \text{circumferential bending stress due to soil load}; \]
\[ s_t = \text{circumferential bending stress due to traffic load}; \]
\[ s_y = \text{yield stress of pipe material}; \]
\[ T = \text{exposure time}; \]
\[ t = \text{pipe wall thickness}; \]
\[ X = \text{random variable}; \]
\[ Z = \text{limit state function}; \]
\[ \alpha = \text{sensitivity coefficient}; \]
\[ \beta = \text{reliability index}; \]
\[ \gamma = \text{unit weight of soil}; \]
\[ \delta = \text{partial change}; \]
\[ \sigma = \text{standard deviation}; \text{ and} \]
\[ \Phi = \text{cumulative distribution function}. \]